Resolution-Based Theorem Proving for Modal and Description Logic

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Introduction

- Aim:
  - To study modal and description logics as fragments of first-order logic
  - To use techniques from first-order resolution for deciding modal and description logics
  - To mention some other applications
- Remarks:
  - Content not as detailed as in ordinary lectures
  - Feel free to ask questions !

Overview

- The logics and translation to FOL
- First-order resolution
- Resolution decision procedures
- Other applications

Part I

The logics and translation to FOL

Basic modal logic

- Basic modal logic $K_{(m)}$ = propositional logic plus $\langle r_1 \rangle, \langle r_2 \rangle, \ldots$
  $Ac = \{r_1, r_2, \ldots\}$ (index set)
- Modal formulae: $\phi, \psi \longrightarrow p_i \mid \neg\phi \mid \phi \vee \psi \mid \langle\alpha\rangle\phi$
  Actions: $\alpha, \beta \longrightarrow r_j$
- $[\alpha]\phi \stackrel{\text{def}}{=} \neg\langle\alpha\rangle\neg\phi$
- Semantics: Kripke model $\mathcal{M} = (W, \{R_j \mid r_j \in Ac\}, v)$

$$
\begin{array}{lll}
\mathcal{M}, x \models p_i & \text{iff} & x \in v(p_i) \\
\mathcal{M}, x \models \neg\phi & \text{iff} & \mathcal{M}, x \not\models \phi \\
\mathcal{M}, x \models \phi \vee \psi & \text{iff} & \mathcal{M}, x \models \phi \text{ or } \mathcal{M}, x \models \psi \\
\mathcal{M}, x \models \langle r_j \rangle\phi & \text{iff} & \text{for some } R_{r_j}\text{-successor } y \text{ of } x \;\; \mathcal{M}, y \models \phi \\
\mathcal{M}, x \models [r_j]\phi & \text{iff} & \text{for all } R_{r_j}\text{-successors } y \text{ of } x \;\; \mathcal{M}, y \models \phi
\end{array}
$$

Extensions of the basic modal logic

- Traditional MLs: extension of $K_{(m)}$ with extra modal axioms
  - epistemic ML, doxastic ML, ...
- Dynamic MLs: extensions of $K_{(m)}$ with operators on actions
  - dynamic logic PDL = $K_{(m)}(\vee, ;, ^*, ?)$
  - description logics with role operators

Reading of $[r_j]\phi$	Notation	Logic
ϕ is necessary	$\Box\phi$	basic modal logic K
agent j knows ϕ	$K_j\phi$	epistemic logic $KT45_{(m)}$
agent j believes ϕ	$B_j\phi$	doxastic logic $KD45_{(m)}$
action r_j causes ϕ	$[r_j]\phi$	dynamic logic PDL
R_j-relatives of only C_ϕs	$\forall R_j.C_\phi$	description logics, $\mathcal{ALC}$ family

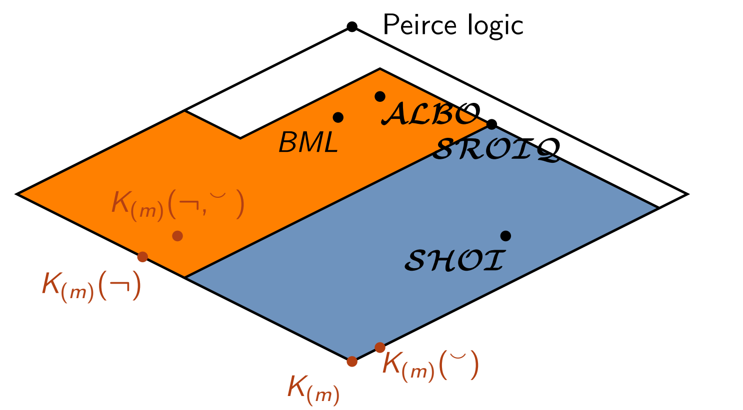
Dynamic modal logics

- $K_{(m)}$ plus action-forming operators
- Actions: $\alpha, \beta \longrightarrow r_j \mid \neg\alpha \mid \alpha \vee \beta \mid \alpha^{\smile} \mid \alpha ; \beta \mid \phi^c \mid id$
- Semantics:

$$
\begin{array}{lll}
R_{\neg\alpha} \stackrel{\text{def}}{=} (W \times W) \setminus R_\alpha & & R_{\alpha\vee\beta} \stackrel{\text{def}}{=} R_\alpha \cup R_\beta \\
R_{\alpha^{\smile}} \stackrel{\text{def}}{=} R_\alpha^{\smile} \stackrel{\text{def}}{=} \{(x,y) \mid (y,x) \in R_\alpha\} & & \\
R_{\alpha ; \beta} \stackrel{\text{def}}{=} \{(x,y) \mid \exists z.(x,z) \in R_\alpha \wedge (z,y) \in R_\beta\} & & \\
R_{\phi^c} \stackrel{\text{def}}{=} \{(x,y) \mid x \in R_\phi\} & & R_{id} \stackrel{\text{def}}{=} Id_W
\end{array}
$$

- ... defines Peirce logic
- Very expressive; undecidable; has many decidable sublogics
- $K_{(m)}(\star_1, \ldots, \star_n) = K_{(m)}$ extended with $\star_1, \ldots, \star_n$

Lattice of dynamic modal logics



decidable DMLs/DLs without relational ¬
decidable DMLs/DLs with relational ¬
this talk

$BML = K_{(m)}(\neg, \vee)$

Why are extensions with ¬ and $^\smile$ interesting?

- $K_{(m)}(^\smile)$ = tense logic; $[\alpha^\smile]$ is past operator
- $K_{(m)}(\neg)$ = the logic of 'some', 'all' and 'only'

possibility op	$\langle\alpha\rangle\phi$	'ϕ is true at some α-successor'
necessity op	$\neg\langle\alpha\rangle\neg\phi$	'ϕ is true at all α-successors'
sufficiency op	$\neg\langle\neg\alpha\rangle\phi$	'ϕ is true at only α-successors'

  - Also definable (by non-logical axioms) are:
    - left cyl., right cyl., cross product, domain/range restriction
- Standard tableau methods decide $K_{(m)}(^\smile)$
- $K_{(m)}(\neg)$ and $K_{(m)}(\neg,^\smile)$ do not have the tree model property
- Using unrestricted blocking $K_{(m)}(\neg)$ and $K_{(m)}(\neg,^\smile)$ can be decided with tableau

First-order logic: Background

- Most important 'unifying' formalism for knowledge representation and reasoning in CS and AI
- Very Expressive: most domain knowledge can be represented with ease
- Sound and complete deduction calculi exist
- Many reasoning tools available
- Reasoning is undecidable (not a problem!)
- Many decidable fragments
- Many practical decision procedures for decidable fragments

Syntax of first-order logic

- Terms:

$s, t, u \longrightarrow x$ (first-order variable)
$\quad\mid\ a$ (constant)
$\quad\mid\ f(s_1, \ldots, s_n),\quad n > 0$ (functional term)

- Atoms:

$A, B \longrightarrow P(s_1, \ldots, s_n),\quad n \geq 0$ (non-equational atom)
$\quad\mid\ s \approx t$ (equational atom)

- First-order formulae:

$F, G \longrightarrow \bot \mid \top$
$\quad\mid\ A$ (atomic formula)
$\quad\mid\ \neg F \mid (F \star G)$ $\quad \star \in \{\wedge, \vee, \rightarrow, \leftrightarrow\}$
$\quad\mid\ \exists x F \mid \forall x F$ (quantified formulae)

Standard translation of MLs into FOL

- Translation mapping: $L \xrightarrow{\Pi} FOL$
  - sound & complete, efficiently computable (linear/polynomial)
- Standard translation based on semantics of source logic L
- Question: $\Gamma \models \phi$?
  Where $\Gamma \stackrel{\text{def}}{=} \{p \rightarrow q\}$ $\quad \phi \stackrel{\text{def}}{=} [r]\langle r\rangle p$
- Equivalent to: $\Pi(\Gamma) \models_{FOL} \Pi(\phi)$?

$$\Pi(\Gamma) = \forall x\,[Q_p(x) \rightarrow Q_q(x)]$$
$$\Pi(\phi) = \forall x \forall y (R(x, y) \rightarrow \exists z (R(y, z) \wedge Q_p(z)))$$

- Now give to any FOL prover

Using translation to FOL

- Let L be given DML/DL
  $F_L \stackrel{\text{def}}{=} \Pi(L)$ corresponding FO fragment
- Π sound & complete $\Rightarrow$ any FOL prover can be used
- Π efficiently computable $\Rightarrow$ if L decidable then F_L decidable
- FO methods are not automatically decision procedures for F_L
  - Identify decidable FO fragment G encompassing F_L and use decision procedure of G
- F_L not necessarily subfragment of known decidable FO fragm.
  - Develop FO decision procedure for F_L
- Decision procedure of G might not be suitable for purpose
  - Develop suitable refinement for purpose of F_L

Part II

First-order resolution

Resolution

- Refutation approach, testing (un)satisfiability
- Operates on clauses
- Two rules: resolution and factoring
- No branching rules required $\rightsquigarrow$ derivations are linear

Resolution: $\dfrac{C \vee A \qquad \neg A \vee D}{C \vee D}$

Factoring: $\dfrac{C \vee A \vee A}{C \vee A}$

N
$\dots$
$\dots$
$\bot$

Theorem 1
Res is sound and (refutationally) complete for propositional and ground clause logic

Clause logic

- Language of resolution is that of clause logic
- Literals:
  $L \longrightarrow A$ (positive literal, atom)
  $\quad\mid\ \neg A$ (negative literal)
- Clauses:
  $C, D \longrightarrow \bot$ (empty clause)
  $\quad\mid\ L_1 \vee \ldots \vee L_k, \quad k \geq 1$ (non-empty clause)
- Free variables interpreted as implicitly universally quantified
- Clauses regarded as multi-sets of literals
  - $P(a) \vee P(a) \vee Q(x)$ is not the same as $P(a) \vee Q(x)$

Transformation to clausal form

- Basic algorithm (too naive):
  1. Transform into prenex normal form (PNF): move quantifiers to the front
     $\leadsto$ $Q_1 x_1 \ldots Q_n x_n G$ (G quantifier-free)
  2. Skolemisation: eliminate quantifiers
     $\leadsto$ quantifer-free formula
  3. Transform into conjunctive normal form (CNF)
     $\leadsto$ $C_1 \wedge \ldots \wedge C_n$
  4. Clausify
     $\leadsto$ set of clauses $N = \{C_1, \ldots, C_n\}$
- For any F: F is satisfiable iff $\mathrm{Cls}(F)$ is satisfiable
- Various standard optimisations exist (see later)

Running example: Transformation to clausal form

- Take $\phi \stackrel{\text{def}}{=} [r](\neg p \vee \langle r \rangle p)$; ϕ is satisfiable in $K_{(m)}$
- FO translation:
  $$\exists x \big[\forall y \, (R(x,y) \rightarrow (\neg Q_p(y) \vee \exists z \, (R(y,z) \wedge Q_p(z))))\big]$$
- Prenex normal form:
  $$\exists x \forall y \exists z \, \big[\neg R(x,y) \vee \neg Q_p(y) \vee (R(y,z) \wedge Q_p(z))\big]$$
- Skolemisation:
  $$\neg R(a,y) \vee \neg Q_p(y) \vee (R(y,f(y)) \wedge Q_p(f(y)))$$
  Sk. const. for $\exists x$ Sk. term for $\exists z$
- CNF:
  $$(\neg R(a,y) \vee \neg Q_p(y) \vee R(y,f(y))) \wedge$$
  $$(\neg R(a,y) \vee \neg Q_p(y) \vee Q_p(f(y)))$$
- Clausal form: drop $\wedge$ and outer (,)

Basic resolution calculus *Res* for FO clause logic

- *Res* for ground clause logic plus unification
- **Resolution:** $\dfrac{C \vee A \qquad \neg B \vee D}{(C \vee D)\sigma}$ if $\sigma = \mathrm{mgu}(A \doteq B)$
- **Factoring:** $\dfrac{C \vee A \vee B}{(C \vee A)\sigma}$ if $\sigma = \mathrm{mgu}(A \doteq B)$
- Example: $\dfrac{Q(y) \vee P(f(y)) \qquad \neg P(z) \vee R(z,a)}{Q(y) \vee R(f(y),a)}$ $\sigma = \{z/f(y)\}$

Theorem 2
Res is sound and (refutationally) complete for FO clause logic

- Problem: Extremely prolific at generating new clauses

Running example (cont'd): Applying basic resolution

- Clausal form:
  1. $\neg R(a,y) \vee \neg Q_p(y) \vee R(y,f(y))$ given
  2. $\neg R(a,y) \vee \neg Q_p(y) \vee Q_p(f(y))$ given
- Resolvents under *Res*:
  3. $\neg R(a,a) \vee \neg Q_p(a) \vee \neg Q_p(f(a)) \vee \neg Q_p(f^2(a))$ (1.3, 2.1)
  4. $\neg R(a,f(y)) \vee R(f(y),f^2(y)) \vee \neg R(a,y) \vee \neg Q_p(y)$ (1.2, 2.3)
  5. $\neg R(a,f^2(y)) \vee R(f^2(y),f^3(y)) \vee \neg R(a,f(y))$ (2.3, 4.4)
     $\vee \neg R(a,y) \vee \neg Q_p(y)$

  etc
- Problem: Termination for satisfiable formulae
  - Clauses expand in width and depth

Modern resolution framework

$\ldots$ = resolution calculus *Res* + restrictions + control

- Guiding principle: Avoid unnecessary inferences whenever possible
- Local restrictions: control inferences performed via
  - Admissible ordering $\succ$
  - Selection function S
- Global restrictions of search space via
  - General notion of redundancy
- Important for implementation: strategies & heuristics, fairness

Local search control parameters

- Admissible ordering $\succ$
  - total, well-founded on ground terms and atoms
  - on ground literals: $\ldots \succ \neg A \succ A \succ \neg B \succ B \succ \ldots$
  - stable under substitutions
- Selection function S: selects only negative literals
  - $S(C)$ = possibly empty multi-set of negative literal occurrences in C
  - Example of selection with selected literals indicated as $\boxed{L}$: $\boxed{\neg A} \lor \neg A \lor B$, $\boxed{\neg B_0} \lor \boxed{\neg B_1} \lor A$
- Idea:
  - Inferences restricted to $\succ$-maximal or $\boxed{S\text{-selected}}$ literals
  - S overrides $\succ$

Ordered resolution calculus with selection $Res_S^{\succ}$

- Assume: $\succ$ admissible atom ordering; S selection function
- **Ordered resolution with selection rule:**

$$\frac{C \lor A \qquad \neg B \lor D}{(C \lor D)\boldsymbol{\sigma}}$$

provided $\boldsymbol{\sigma} = \text{mgu}(A \doteq B)$ and

(i) $A\boldsymbol{\sigma}$ strictly maximal wrt. $C\boldsymbol{\sigma}$;

(ii) nothing selected in C by S;

(iii) either $\neg B$ selected,
or else nothing selected in $\neg B \lor D$
and $\neg B\boldsymbol{\sigma}$ maximal wrt. $D\boldsymbol{\sigma}$

- Note: variables of premises must be renamed apart

Ordered resolution calculus with selection $Res_S^{\succ}$ (cont'd)

- **Ordered factoring rule:**

$$\frac{C \lor A \lor B}{(C \lor A)\boldsymbol{\sigma}}$$

provided $\boldsymbol{\sigma} = \text{mgu}(A \doteq B)$ and

(i) $A\boldsymbol{\sigma}$ is maximal wrt. $C\boldsymbol{\sigma}$;

(ii) nothing is selected in C

Theorem 3
$Res_S^{\succ}$ is sound and (refutationally) complete for FO clause logic

Search spaces become smaller

- Assume $P \succ Q \succ R \succ T$ and nothing is selected, i.e. $S = \emptyset$

1. $\neg T \vee \underline{P} \vee Q$	given
2. $\underline{\neg P} \vee \neg R$	given
3. $\underline{\neg Q}$	given
4. $\neg T \vee \underline{Q} \vee \neg R$	Res 1, 2
5. $\neg T \vee \underline{\neg R}$	Res 3, 4

- Derivation is completely deterministic
- Generally, proof search still non-deterministic but search space is much smaller than with unrestricted resolution
- Exercise: Choose selection function so that no inferences are possible

Part III

Decision procedures

Running example (cont'd): Using ordered resolution

- Recall using *Res* clauses expand in width and depth
- Use ordering and/or selection function to prevent this

1. $\neg R(a, y) \vee \neg Q_p(y) \vee R(y, f(y))$	given
2. $\neg R(a, y) \vee \neg Q_p(y) \vee Q_p(f(y))$	given

- Let $\succ$ extension of subterm ordering + no selection f. ($S = \emptyset$)
  - $f(t) \succ t$; precedence on pred. symbols: $R \succ Q_p$
  - first criterion: $\succ$ on maximal arguments
- No inference steps possible in $Res^{\succ}$!

1. $\neg R(a, y) \vee \neg Q_p(y) \vee \underline{R(y, f(y))}$	given
2. $\neg R(a, y) \vee \neg Q_p(y) \vee \underline{Q_p(f(y))}$	given

Decidability of $K_{(m)}$ by ordered resolution

- How to show that $Res^{\succ}$ decides $K_{(m)}$?
  - Characterise a class of clauses closed under $Res^{\succ}$ into which any $K_{(m)}$-problem can be mapped
  - Show the class is bounded when defined over a bounded signature of predicate and function symbols
- Required: structural transformation ...

Structural transformation of first-order formulae

Theorem 4
Let Q be a fresh predicate symbol. Then

$$F[G(\overline{x})] \text{ satisf. iff } F[Q(\overline{x})] \wedge \forall \overline{x}(Q(\overline{x}) \leftrightarrow G(\overline{x})) \text{ satisf.}$$

- **Structural transformation rewrite rule**:

$$F[G(\overline{x})] \;\Rightarrow\; F[Q(\overline{x})] \wedge \forall \overline{x}(Q(\overline{x}) \leftrightarrow G(\overline{x}))$$

  - Introduces new pred. symbol Q for subformula $G(\overline{x})$ of F
  - View $Q(\overline{x})$ as an abbreviation for $G(\overline{x})$.
- Small overhead; efficient transformation to CNF
- Our case: Introduce new Q_ϕ $\forall$ non-negated complex ϕ
  Take polarity of subformulae into account

Structural transformation for running example

- FO translation of $\phi = [r](\neg p \vee \langle r\rangle p)$:

$$\exists x \left[\underbrace{\forall y\, (\neg R(x,y) \vee \underbrace{(\neg Q_p(y) \vee \underbrace{\exists z\, (R(y,z) \wedge Q_p(z))}_{Q_{\langle r\rangle p}(y)})}_{Q_{\neg p \vee \langle r\rangle p}(y)}}_{Q_{\forall r.(\neg p \vee \langle r\rangle p)}(x)} \right]$$

- Clausal form $\mathrm{Cls}\,\Xi(\neg\Pi(\phi))$:

$$\neg Q_{\langle r\rangle p}(x) \vee R(x, f(x))$$
$$\neg Q_{\langle r\rangle p}(x) \vee Q_p(f(x))$$
$$\neg Q_{\neg p \vee \langle r\rangle p}(x) \vee \neg Q_p(x) \vee Q_{\langle r\rangle p}(x)$$
$$\neg Q_{[r](\neg p \vee \langle r\rangle p)}(x) \vee \neg R(x,y) \vee Q_{\neg p \vee \langle r\rangle p}(y)$$
$$Q_{[r](\neg p \vee \langle r\rangle p)}(a)$$

General form of input clauses

- Form of input clauses for $K_{(m)}$:

$$(\neg)Q_\phi(a)$$
$$R(a,b)$$
$$(\neg)Q_\phi(x) \vee (\neg)Q_1(x) \vee \ldots \vee (\neg)Q_n(x)$$
$$(\neg)Q_\phi(x) \vee \neg R(x,y) \vee (\neg)Q(y)$$
$$(\neg)Q_\phi(x) \vee R(x, f_\phi(x))$$
$$(\neg)Q_\phi(x) \vee (\neg)Q(f_\phi(x))$$

- Ordering: binary literals $\succ$ unary literals
  depth 2 literals $\succ$ depth 1 literals
- Step 1: In each clause what are the maximal literals?

General form of input clauses : Maximal literals

- Form of input clauses for $K_{(m)}$:

$$\underline{(\neg)Q_\phi(a)}$$
$$\underline{R(a,b)}$$
$$(\neg)Q_\phi(x) \vee \underline{(\neg)Q_1(x)} \vee \ldots \vee (\neg)Q_n(x) \quad (\geq 1 \text{ max. lits})$$
$$(\neg)Q_\phi(x) \vee \underline{\neg R(x,y)} \vee (\neg)Q(y)$$
$$(\neg)Q_\phi(x) \vee \underline{R(x, f_\phi(x))}$$
$$(\neg)Q_\phi(x) \vee \underline{(\neg)Q(f_\phi(x))}$$

- Ordering: binary literals $\succ$ unary literals
  depth 2 literals $\succ$ depth 1 literals
- Step 1: In each clause what are the maximal literals?
- Step 2: What do the resolvents & factors look like?

Clausal class *MC*

- General form of derived clauses
  - ground clauses with only unary literals
  - $(\neg)Q_\phi(x) \vee (\neg)Q_1(x) \vee \ldots \vee (\neg)Q_n(x) \qquad (0 \leq n)$
  - $(\neg)Q_\phi(x) \vee (\neg)Q_1(x) \vee \ldots \vee (\neg)Q_n(x)$
    $\vee (\neg)Q_1(f_\phi(x)) \vee \ldots \vee (\neg)Q_m(f_\phi(x)) \quad (0 \leq n, m)$
- Let MC = class of these clauses:
  - ground unary clauses
  - $R(a, b)$
  - non-ground unary clauses with arguments x or $f_\phi(x)$
  - $(\neg)Q_\phi(x) \vee \neg R(x, y) \vee (\neg)Q(y)$
  - $(\neg)Q_\phi(x) \vee R(x, f_\phi(x))$

Decidability of $K_{(m)}$ by ordered resolution

Lemma 5
For any finite clause set N in MC:

1. Any derived clause belongs to MC
2. Any $Res^{\succ}$-derivation from N terminates in EXPTIME

Theorem 6
Assume ϕ any formula and any set Γ in $K_{(m)}$;
let $N = \mathrm{Cls}\,\Xi(\Pi(\Gamma) \wedge \neg\Pi(\phi))$

1. Any $Res^{\succ}$-derivation from N terminates in EXPTIME
2. $\Gamma \models \phi$ iff $Res^{\succ}$ derives $\bot$ from N

- Complexity is optimal for $\Gamma \neq \emptyset$

Generalisation

- Clausal class *MC* :
  - ground unary clauses
  - $R(a, b)$
  - non-ground unary clauses with arguments x or $f(x)$
  - $(\neg)Q_\phi(x) \vee \neg R(x, y) \vee (\neg)Q(y)$
  - $(\neg)Q_\phi(x) \vee R(x, f_\phi(x))$
- What if binary literals are negated ?

Generalisation

- Clausal class extended :
  - ground unary clauses
  - $(\neg)R(a, b)$
  - non-ground unary clauses with arguments x or $f(x)$
  - $(\neg)Q_\phi(x) \vee (\neg)R(x, y) \vee (\neg)Q(y)$
  - $(\neg)Q_\phi(x) \vee (\neg)R(x, f_\phi(x))$
- Lemma true for the extended class
- Thus, the theorem is true for $K_{(m)}(\neg)$!
- What if arguments in binary literals can be swapped ?

Generalisation

- Clausal class MC^* :
  - ground unary clauses
  - $(\neg)\underline{R^{(\smile)}(a, b)}$
  - non-ground unary clauses with arguments x or $f(x)$
  - $(\neg)Q_\phi(x) \vee (\neg)\underline{R^{(\smile)}(x, y)} \vee (\neg)Q(y)$
  - $(\neg)Q_\phi(x) \vee (\neg)\underline{R^{(\smile)}(x, f_\phi(x))}$
- Lemma true for the extended class
- Thus, the theorem is true for $K_{(m)}(\neg)$!
- And for $K_{(m)}(^\smile)$ and $K_{(m)}(\neg, ^\smile)$!

Ordered resolution decides $K_{(m)}(\neg, ^\smile)$

Theorem 7

$Res^{\succ}$ is decision procedure for any logic between $K_{(m)}$ and $K_{(m)}(\neg, ^\smile)$ and has (optimal) EXPTIME complexity

- Also true for any logic between $K_{(m)}$ and $K_{(m)}(\neg, ^\smile, \upharpoonright, \downharpoonright, \cdot^c, {}^c\cdot, \times)$
- Using the axiomatic translation translation many traditional MLs, incl. *KD45*, *S4*, ..., can be efficiently embedded into MC^*
- Gives complexity optimal decision procedures

Generalisation to *BML* and beyond

- Ordered resolution decides wider clausal class: DL^*

$MC^* \subseteq DL^*$	$MC^* \subseteq DL^*$
$BML \subseteq DL^*$	$BML(^\smile, ;^{pos}) \subseteq DL^*$
$\mathrm{FO}^2 \subseteq DL^*$	$\mathrm{FO}^3 \cap DL^* \neq \emptyset$

- DL^* subsumes many DLs
- DL^* is NEXPTIME-complete

Theorem 8

$Res^{\succ}$ + condensing, or splitting, decides DL^*, and hence all subsumed logics, incl. *BML* and $BML(^\smile, ;^{pos})$

Generalisation to decidable fragments of FOL

- Numerous ways of defining decidable subclasses of FOL

Restrict ...	Decidable classes
arity of predicate symbols	monadic class
quantifier prefixes	$\exists^*\forall^*$, $\exists^*\forall\exists^*$, $\exists^*\forall\forall\exists^*$
number of variables	FO^2
ordering on variables	fluted logic
quantification by relativisation	guarded fragments
$\forall$ quantification	Maslov's dual class $\overline{K}$, $\overline{DK}$

- All decidable by resolution (with 1 exception based on extensions of $Res^{\succ}$)

Part IV

Other applications and conclusion

Some other applications

- Simulating, generating, implementing and studying different deduction approaches (Thursday)
- Automatically generating models, incl. minimal models
- Second-order quantifier elimination
  - Reasoning with second-order formulae (e.g. modal axioms, rules)
  - Automatically computing correspondence properties

Automated correspondence theory

- Given: traditional ML with extra axioms/rules, e.g. $K_{(m)}\Delta$
  Problem: What are first-order frame correspondence properties for axioms/rules in Δ?
- Second-order quantifer elimination methods solve the problem
  - E.g. SCAN (based on resolution)
  - $\forall p[\Box p \longrightarrow \Box\Box p]$ $\rightsquigarrow$ transitivity of R
- Main issue: successful termination
  - SCAN solves problem for all Sahlqvist formulae and inductive formulae
  - Automatic solution possible for even wider class
- New book: *Second-Order Quantifier Elimination* by Gabbay, Schmidt & Szałas, College Publ., 2008.

Concluding remarks

- Combination of translation and resolution has practical and theoretical advantages
- Translation is a core technique in computer science
- Resolution provides a powerful and versatile framework
  - for developing practical decision procedures
  - for other applications
- Well-developed implementation: SPASS 3.5

Part V

Selected references

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